

Bounce and cyclic cosmology in weakly broken galileon theories

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ABSTRACT: We investigate the bounce and cyclicity realization in the framework of weakly broken galileon theories. We study bouncing and cyclic solutions at the background level, reconstructing the potential and the galileon functions that can give rise to a given scale factor, and presenting analytical expressions for the bounce requirements. We proceed to a detailed investigation of the perturbations, which after crossing the bouncing point give rise to various observables, such as the scalar and tensor spectral indices and the tensor-to-scalar ratio. Although the scenario at hand shares the disadvantage of all bouncing models, namely that it provides a large tensor-to-scalar ratio, introducing an additional light scalar significantly reduces it through the kinetic amplification of the isocurvature fluctuations.

KEYWORDS: Modified gravity, Galileon symmetry, Bounce, Cyclic cosmology

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1 Introduction

Inflation is now considered to be a crucial part of the universe cosmological history [1], however the so called “standard model of the universe” still faces the problem of the initial singularity. Such a singularity is unavoidable if inflation is realized using a scalar field while the background spacetime is described by the standard Einstein action [2]. As a consequence, there has been a lot of effort in resolving this problem through quantum gravity effects or effective field theory techniques.

A potential solution to the cosmological singularity problem may be provided by non-singular bouncing cosmologies [3]. Such scenarios have been constructed through various approaches to modified gravity [4, 5], such as the Pre-Big-Bang [6] and the Ekpyrotic [7, 8] models, gravity actions with higher order corrections [9, 10], $f(R)$ gravity [11, 12], $f(T)$ gravity [13], braneworld scenarios [14, 15], non-relativistic gravity [16, 17], massive gravity [18], Lagrange modified gravity [19], loop quantum cosmology [20–22] or in the frame of a closed universe [23]. Non-singular bounces may be alternatively investigated using effective field theory techniques, introducing matter fields violating the null energy condition [24–26], or introduce non-conventional mixing terms [27, 28]. The extension of all the above bouncing scenarios is the (old) paradigm of cyclic cosmology [29], in which the universe experiences the periodic sequence of contractions and expansions, which has been reawaked the last years [30, 31] since it brings different insights for the origin of the observable universe [32–34] (see [35] for a review). Such scenarios are also capable of explaining the

scale invariant power spectrum [35, 36] and moderate non-Gaussianities [37]. Hence, they are considered as a potential alternative to Big Bang cosmology.

One very general class of gravitational modification are galileon theories [38–40], which are a re-discovery of Horndeski general scalar-tensor theory [41], in which one introduces higher derivatives in the scalar-tensor action, with the requirement of maintaining the equations of motion second-ordered. In this formulation the Lagrangian is imposed to satisfy the Galilean symmetry $\phi \rightarrow \phi + b_\mu x^\mu$, with b_μ a constant, and an additional advantage is that the scalar field derivative self-couplings screen the deviations from General Relativity at high gradient regimes due to the Vainshtein mechanism [42], thus satisfying the solar system constraints. These features led galileon theories and their modifications to have an extensive application in cosmological frameworks. In particular, one can study the late-time acceleration [43–47], inflation [48–50] and non-Gaussianities [51–53], cosmological perturbations [54–56], and use observational data to constrain various classes of galileon theories [57–59].

Recently, a model of weakly broken galileon symmetry appeared in the literature [60]. In this construction the notion of weakly broken galileon invariance was introduced, which characterizes the unique class of gravitational couplings that maximally preserve the defining symmetry. Hence, the curved-space remnants of the quantum properties of the galileon allow one to construct quasi de Sitter backgrounds that remain to a large extent insensitive to loop corrections [60].

In the present work, we are interested in investigating the bounce and cyclicity realization in the framework of weakly broken galileon theories. Although the bouncing realization has been shown to be possible in the context of usual galileon cosmology [61–64], we show that in the present weakly broken variance we have enhanced freedom to satisfy the relevant requirements. The plan of the work is as follows: In Section 2 we briefly review theories with weakly broken galileon invariance, and we apply them in a cosmological framework. In Section 3 we investigate the realization of bouncing and cyclic solutions at the background level, reconstructing the corresponding potentials and the galileon functions. In Section 4 we analyze the perturbations of the scenario, and we study how they pass through the bouncing point, giving rise to various observables, such as the scalar and tensor spectral indices and the tensor-to-scalar ratio. Finally, in section 5 we summarize our results.

2 Cosmology with weakly broken galileon symmetry

Let us briefly review theories with weakly broken galileon invariance following [60]. Such constructions include a scalar field coupled to gravity, and form a subclass of Horndeski theories which only weakly breaks the galileon symmetry even in the presence of gravity. This property is achieved by suitably formulating these theories in order for the symmetry-breaking interaction terms in the Lagrangian to be suppressed. The advantage of this procedure is that the resulting field equations remain of second order, although the Lagrangian includes higher derivative interaction terms.

The action of this class of theories reads as [60]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{pl}^2 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \sum_{I=2}^5 \mathcal{L}_I^{\text{WBG}} + \dots \right] + S_m , \quad (2.1)$$

with ϕ the scalar field, R the Ricci scalar, M_{pl} the Planck mass, S_m the matter-sector action, and where we have defined the operators $\mathcal{L}_I^{\text{WBG}}$ to be given by the following subclass of the Horndeski terms:

$$\mathcal{L}_2^{\text{WBG}} = \Lambda_2^4 G_2(X) , \quad (2.2)$$

$$\mathcal{L}_3^{\text{WBG}} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X) [\Phi] , \quad (2.3)$$

$$\mathcal{L}_4^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X) R + 2 \frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X) ([\Phi]^2 - [\Phi^2]) , \quad (2.4)$$

$$\mathcal{L}_5^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^9} G_5(X) G_{\mu\nu} \Phi^{\mu\nu} - \frac{\Lambda_2^4}{3\Lambda_3^9} G_{5X}(X) ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]) . \quad (2.5)$$

In the above expressions G_I are arbitrary dimensionless functions of the dimensionless variable

$$X \equiv -\frac{1}{\Lambda_2^4} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi , \quad (2.6)$$

and we have used the subscript “ X ” to denote differentiation with respect to this variable, while $G_{\mu\nu}$ is the Einstein tensor. Furthermore, we have introduced the compact notation [60]

$$\begin{aligned} [\Phi] &\equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \phi \\ [\Phi^2] &\equiv \nabla^\mu \nabla_\nu \phi \nabla^\nu \nabla_\mu \phi \\ &\dots \end{aligned} \quad (2.7)$$

Additionally, the parameter Λ_3 marks the scale suppressing the invariant galileon interactions, while the parameter $\Lambda_2 = (M_{pl} \Lambda_3^3)^{1/4}$, with $\Lambda_3 \ll \Lambda_2$, marks the significantly higher scale suppressing the quantum-mechanically generated single-derivative operators [60]. Obviously, in the limit where both Λ_2, Λ_3 go to M_{pl} , weakly broken galileon invariance disappears, and the above theories become the usual covariant galileon ones. Note that in action (2.1) one can consider a potential $V(\phi)$, which is the only term that breaks the scalar shift symmetry, which is otherwise exact even in curved space.

Let us now apply the above theories in a cosmological framework. In particular, we consider a flat Friedmann-Robertson-Walker (FRW) spacetime metric of the form

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j , \quad (2.8)$$

where $a(t)$ is the scale factor. For this metric, the metric field equations derived from action (2.1) become the two Friedmann equations [60]

$$3M_{pl}^2 H^2 = \rho_m + V + \Lambda_2^4 X \left[\frac{1}{2} - \frac{G_2}{X} + 2G_{2X} - 6ZG_{3X} - 6Z^2 \left(\frac{G_4}{X^2} - 4\frac{G_{4X}}{X} - 4G_{4XX} \right) + 2Z^3 \left(5\frac{G_{5X}}{X} + 2G_{5XX} \right) \right], \quad (2.9)$$

$$M_{pl}^2 \dot{H} = - \frac{\Lambda_2^4 X F + M_{pl} \ddot{\phi} (XG_{3X} - 4ZG_{4X} - 8ZXG_{4XX} - 3Z^2 G_{5X} - 2Z^2 XG_{5XX})}{1 + 2G_4 - 4XG_{4X} - 2ZXG_{5X}} - \frac{\rho_m}{2} - \frac{p_m}{2}, \quad (2.10)$$

with ρ_m and p_m the energy density and pressure of the matter sector, assumed to correspond to a perfect fluid, and where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes differentiation with respect to t . In the above expressions we have defined the function

$$F(X, Z) = \frac{1}{2} + G_{2X} - 3ZG_{3X} + 6Z^2 \left(\frac{G_{4X}}{X} + 2G_{4XX} \right) + Z^3 \left(3\frac{G_{5X}}{X} + 2G_{5XX} \right), \quad (2.11)$$

with the variable Z defined as

$$Z \equiv \frac{H\dot{\phi}}{\Lambda_3^3}. \quad (2.12)$$

Additionally, the equation of motion for the scalar field becomes [60]

$$\frac{1}{a^3} \frac{d}{dt} \left[2a^3 \dot{\phi} F(X, Z) \right] = - \frac{dV}{d\phi}. \quad (2.13)$$

Finally, note that according to definition (2.6), in FRW geometry we have $X = \dot{\phi}^2/\Lambda_2^4$. Lastly, note that the above equations close considering the matter conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \quad (2.14)$$

3 Background bouncing and cyclic solutions

In this section we are interested in investigating the bounce and cyclicity realization in cosmologies with weakly broken galileon invariance, at the background level. Let us first review the basic conditions for these realizations. An expanding universe is characterized by a positive Hubble parameter, while a contracting one by a negative H . Using the continuity equations we deduce that at the bounce and turnaround points $H = 0$. However, at and around the bounce we must have $\dot{H} > 0$, while at and around the turnaround we obtain $\dot{H} < 0$.

One can easily see that the above conditions cannot be fulfilled in the framework of general relativity, nevertheless they can be easily satisfied in the scenario at hand. In particular, observing the form of the two Friedmann equations (2.9), (2.10), along with the scalar-field equation (2.13), we conclude that for suitable choices of the free functions G_I and of the scalar potential $V(\phi)$ one can acquire the necessary violation of the null energy condition and hence the satisfaction of the bouncing and cyclic conditions.

Let us make an important point concerning the bounce and cyclicity reconstruction. In principle, in the present scenario, one has the freedom to determine the free functions G_I 's as well as the scalar potential $V(\phi)$. However, note that while in the case where the G_I 's are all zero (i.e in the case of minimally-coupled general relativity) there is no potential that can drive a bounce, in the case of suitably chosen non-trivial G_I 's a bounce can be realized either with a zero potential or with a suitably chosen non-zero potential. From these we deduce that the crucial ingredient of bounce and cyclicity realization is the Galileon functions G_I 's and not the potential $V(\phi)$. This feature, along the fact that in Galileon construction shift symmetry plays a crucial role and thus a potential is absent, led the initial works of Galileon cosmology, and Galileon bouncing cosmology in particular, not to consider a scalar potential and focus on the special choice of the G_I 's functions [61, 62]. Nevertheless, since in the generalized Galileon theory (or in the point of view of Horndeski theory), which is the basis of the present work, a scalar potential is allowed, one has an additional free function to play with, and thus he can alleviate some tuning from the functions G_I 's.

Hence, in the following subsections, for completeness, we will reconstruct bounce and cyclic cosmology reconstructing first the necessary scalar potential for a not so tuned choice of the functions G_I 's, and then reconstructing G_I 's for a zero or non-zero given potential.

3.1 Reconstruction of a bounce

Let us now present the bounce realization at the background level. Without loss of generality we consider a bouncing scale factor of the form

$$a(t) = a_b(1 + Bt^2)^{1/3}, \quad (3.1)$$

where a_b is the scale factor value at the bounce, while B is a positive parameter which determines how fast the bounce takes place. In this case time varies between $-\infty$ and $+\infty$, with $t = 0$ the bouncing point. Hence, since the scale factor is known we can straightforwardly find the forms of $H(t)$ and $\dot{H}(t)$ as

$$H(t) = \frac{2Bt}{3(1 + Bt^2)} \quad (3.2)$$

$$\dot{H}(t) = \frac{2B}{3} \left[\frac{1 - Bt^2}{(1 + Bt^2)^2} \right]. \quad (3.3)$$

As we discussed above, one can realize the above background bouncing solution either choosing (without tuning) the forms of all the functions G_I 's and suitably reconstruct the scalar potential $V(\phi)$, or choose a zero or a simple $V(\phi)$ and some of the G_I 's and suitably reconstruct the remaining G_I . In the following we investigate these two procedures separately.

3.1.1 Reconstructing $V(\phi)$

Let us first study the case where the ansatzes for the functions G_I 's are considered by hand, without any particular form of tuning. According to the discussion in [60], in theories with

weakly broken galileon invariance the functions G_2 and G_4 should be assumed to start at least quadratic in X . Hence, the simplest class of models with weakly broken galileon symmetry would be

$$G_2 = G_4 = X^2; \quad G_3 = X; \quad G_5 = 0. \quad (3.4)$$

Inserting (3.1) and (3.4) into the Friedmann equations (2.9),(2.10) we obtain

$$3M_{pl}^2 H(t)^2 = \rho_m(t) + V(\phi(t)) + \dot{\phi}(t)^2 \left[\frac{1}{2} + \frac{3\dot{\phi}(t)^2}{\Lambda_2^4} - \frac{6H(t)\dot{\phi}(t)}{\Lambda_3^3} + \frac{90H(t)^2\dot{\phi}(t)^2}{\Lambda_3^6} \right] \quad (3.5)$$

$$\left[M_{pl}^2 \dot{H}(t) + \frac{\rho_m(t)}{2} + \frac{p_m(t)}{2} \right] \left[1 - \frac{\dot{\phi}(t)^4}{\Lambda_2^8} \right] = M_{pl} \frac{\dot{\phi}(t)^2}{\Lambda_2^4} \left[1 - 24 \frac{H(t)\dot{\phi}(t)}{\Lambda_3^3} \right] \ddot{\phi}(t) - \dot{\phi}(t)^2 F(\dot{\phi}(t)), \quad (3.6)$$

while using (2.11) the function $F(X, Z)$ reads as

$$F(\dot{\phi}(t)) = \frac{1}{2} + 2 \frac{\dot{\phi}^2}{\Lambda_2^4} - \frac{3H\dot{\phi}}{\Lambda_3^3} + 36 \frac{H^2 \dot{\phi}^2}{\Lambda_3^6}. \quad (3.7)$$

Similarly, the scalar-field equation (2.13) becomes

$$\frac{1}{a(t)^3} \frac{d}{dt} \left[2a(t)^3 \dot{\phi}(t) F(\dot{\phi}(t)) \right] = - \frac{\dot{V}(\phi(t))}{\dot{\phi}(t)}. \quad (3.8)$$

Note that we have considered all quantities in the above equations to depend on t , and $a(t)$, $H(t)$, $\dot{H}(t)$ are given by (3.1),(3.2),(3.3).

As we can see, the second Friedmann equation (3.6) is independent of the potential $V(\phi(t))$. Hence, once the matter equation-of-state parameter is given (in which case (2.14) provides $\rho_m(t)$), Eq. (3.6) can be used to provide a solution for $\phi(t)$ and $\dot{\phi}(t)$. In particular, Eq. (3.6) is a simple differential equation for $\dot{\phi}(t)$, namely

$$\ddot{\phi}(t) = Q(\dot{\phi}(t), t), \quad (3.9)$$

that can be easily solved to find $\dot{\phi}(t)$ and hence $\phi(t)$. Similarly, the scalar-field equation (3.8) is a simple differential equation for $V(t)$ of the form

$$\dot{V}(t) = P(\dot{\phi}(t), t). \quad (3.10)$$

Thus, substituting the solution for $\dot{\phi}(t)$ into (3.10) and integrating we can immediately find $V(t)$. In summary, having found the solution for $\phi(t)$ and $V(t)$ we can obtain $V(\phi)$ in a parametric form. Hence, this re-constructed potential will be the one that generates the bouncing scale factor (3.1). As we described above, we mention that the freedom to have a potential allows to obtain a bounce even for simple and not tuned G_I 's, as those chosen in (3.4), which is the motivation of the present paragraph.

In general the above procedure cannot be performed analytically, due to the complicated forms of the involved equations. Therefore, in order to provide a concrete example, we proceed to a numerical application of the above steps. Moreover, since we desire to

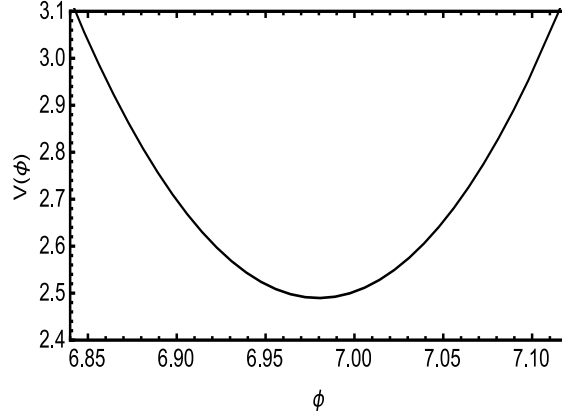


Figure 1. The reconstructed scalar potential $V(\phi)$ that generates the bouncing scale factor (3.1), in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. The bouncing parameters have been chosen as $a_b = 0.2$, $B = 10^{-5}$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

investigate the pure effect of the novel terms of action (2.1), we neglect the matter sector. In Figure 1 we present the potential $V(\phi)$ that is reconstructed from the given bouncing scale-factor form (3.1), according to the above procedure.

As we can see from Figure 1, in order to obtain a bouncing scale factor in the case where $G_2 = G_4 = X^2$, $G_3 = X$, we need a potential with a simple minimum. Hence, we can now reverse the reconstruction procedure and consider a potential of the simple form

$$V(\phi) = V_0 + (\phi - \phi_0)^2, \quad (3.11)$$

where V_0 and ϕ_0 are parameters. Inserting this form into Eqs. (3.5) and (3.8), we obtain a system of two ordinary differential equations for $a(t)$ and $\phi(t)$, that can be easily solved numerically. In Figure 2 we depict the scale factor $a(t)$ that results from the given potential (3.11). Hence, we indeed verify that the simple parabolic potential (3.11) can generate a cosmological bounce. We mention that the above procedures can be straightforwardly

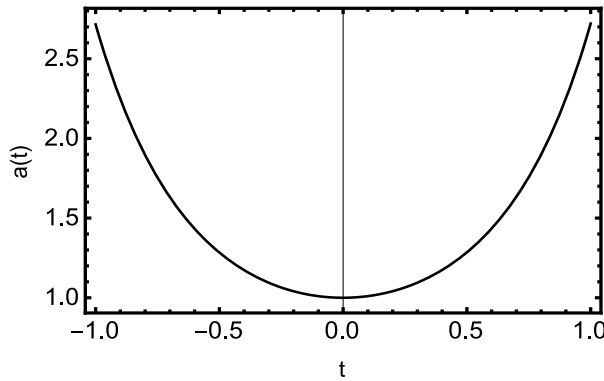


Figure 2. The evolution of the scale factor $a(t)$ that is generated by the simple parabolic potential (3.11), in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. The potential parameters have been chosen as $V_0 = 8.5$, $\phi_0 = 7.0$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

applied in the case where the matter sector is present, i.e describing a matter bounce. In particular, one can repeat the above steps, with the inclusion of a pressureless matter, i.e. with $p_m = 0$ and $\rho_{m0} = \rho_{m0}/a^3$, where ρ_0 is the matter energy density at the time of the bounce.

3.1.2 Reconstructing $G_3(X)$

In this paragraph we chose a priori three out of the four G_I 's, and we set the potential to be zero or to have a specific given form, and we suitably reconstruct the remaining G_I in order to obtain the bouncing solution (3.1). Without loss of generality we determine by hand G_2 , G_4 and G_5 without any particular tuning, i.e we chose

$$G_2 = G_4 = X^2; \quad G_5 = 0, \quad (3.12)$$

and furthermore we assume that the potential $V(\phi)$ is determined too. Inserting (3.1) and (3.12) into the Friedmann equations (2.9),(2.10) we obtain

$$3M_{pl}^2 H(t)^2 = \rho_m(t) + V(\phi(t)) + \dot{\phi}(t)^2 \left[\frac{1}{2} + \frac{3\dot{\phi}(t)^2}{\Lambda_2^4} - \frac{6H(t)\dot{\phi}(t)}{\Lambda_3^3} G_{3X}(t) + \frac{90H(t)^2 \dot{\phi}(t)^2}{\Lambda_3^6} \right] \quad (3.13)$$

$$\left[M_{pl}^2 \dot{H}(t) + \frac{\rho_m(t)}{2} + \frac{p_m(t)}{2} \right] \left[1 - \frac{6\dot{\phi}(t)^4}{\Lambda_2^8} \right] = M_{pl} \frac{\dot{\phi}(t)^2}{\Lambda_2^4} \left[G_{3X}(t) - 24 \frac{H(t)\dot{\phi}(t)}{\Lambda_3^3} \right] \ddot{\phi}(t) - \dot{\phi}(t)^2 F(\dot{\phi}(t)), \quad (3.14)$$

while using (2.11) the function $F(X, Z)$ becomes

$$F(\dot{\phi}(t)) = \frac{1}{2} + \frac{2\dot{\phi}(t)^2}{\Lambda_2^4} - \frac{3H(t)\dot{\phi}(t)}{\Lambda_3^3} G_{3X}(t) + \frac{36H(t)^2 \dot{\phi}(t)^2}{\Lambda_3^6}. \quad (3.15)$$

Similarly, the scalar-field equation (2.13) writes as

$$\frac{1}{a(t)^3} \frac{d}{dt} \left[2a(t)^3 \dot{\phi}(t) F(\dot{\phi}(t)) \right] = -\frac{dV(\phi)}{d\phi}(t). \quad (3.16)$$

Note that we have considered all quantities in the above equations to depend on t , and $a(t)$, $H(t)$, $\dot{H}(t)$ are given by (3.1),(3.2),(3.3).

Equations (3.13), (3.14) and (3.16), out of which only two are independent, form a system of differential equations for $\dot{\phi}(t)$ and $G_{3X}(t)$. Eq. (3.13) can be immediately algebraically solved in terms of $G_{3X}(t)$, and thus insertion into (3.15) and then into (3.16), once the matter equation-of-state parameter is given, leads to a simple second-order differential equation for $\phi(t)$, namely

$$\ddot{\phi}(t) = S(\dot{\phi}(t), \phi(t), t), \quad (3.17)$$

that can be easily solved to find $\phi(t)$. Then, insertion of $\phi(t)$ back in (3.13) gives $G_{3X}(t)$. Finally, knowing $\phi(t)$, i.e $\dot{\phi}(t)$, i.e $X(t)$, as well as $G_{3X}(t)$ allows us to reconstruct $G_{3X}(X)$ and by integration $G_3(X)$. Note that the above procedure is significantly simplified in the

case where the potential is absent, since then (3.17) becomes an algebraic equation for $\dot{\phi}$, namely

$$a(t)^3 \dot{\phi}(t) \left[\frac{1}{4} + \frac{\dot{\phi}(t)^2}{2\Lambda_2^4} - \frac{9H(t)^2 \dot{\phi}(t)^2}{\Lambda_3^6} + \frac{M_{pl}^2 H(t)^2 - \rho_m(t)}{2\dot{\phi}(t)^2} \right] = const., \quad (3.18)$$

which then leads to an easy determination of $G_{3X}(t)$.

In summary, the above procedure allows us to reconstruct $G_3(X)$, which will be the one that generates the bouncing scale factor (3.1). As we described above, we mention that fixing the potential or taking it to be zero, leads to a complicated form of one of the G_I 's (in our example of G_3) in order for the bouncing solution to be realized.

Hence, one can now clearly see the difference in the procedure of the present paragraph, with that of paragraph 3.1.1. In the present analysis the bounce is obtained with a simple or zero potential but with a suitably reconstructed, complicated G_3 , while in paragraph 3.1.1 the bounce was obtained with simple G_I 's, but with a suitably reconstructed, complicated potential.

In general the above procedure cannot be performed analytically, due to the complicated forms of the involved equations. Therefore, in order to provide a concrete example, we proceed to a numerical application of the aforementioned steps. Furthermore, since we desire to investigate the pure effect of the novel terms of action (2.1), we neglect the matter sector. In Fig. 3 we present the function $G_3(X)$ that is reconstructed from the given bouncing scale-factor form (3.1), according to the above procedure, in the case where the scalar potential is zero. As we can see, even in the case of zero potential, with simple choices for the three G_I 's, a bounce can still be realized if one uses a suitably reconstructed, complicated galileon function G_3 .

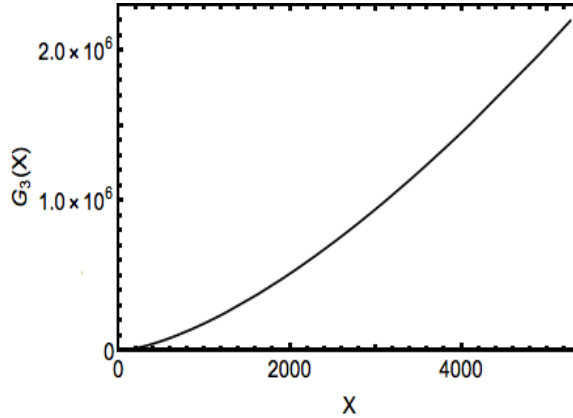


Figure 3. The reconstructed galileon function $G_3(X)$ that generates the bouncing scale factor (3.1), in the case where $V(\phi) = 0$, and with $G_2 = G_4 = X^2$, $G_5 = 0$. The bouncing parameters have been chosen as $a_b = 0.2$, $B = 10^{-5}$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

3.1.3 Analytical conditions for bouncing solutions

We close this subsection by investigating analytical bouncing solutions in the case of matter absence. In particular, substituting (2.12) into the first Friedmann equation (2.9) we obtain

the general equation satisfied by the Hubble function, namely

$$aH^3 + bH^2 + cH + d = 0, \quad (3.19)$$

where a, b, c, d are time-dependent coefficients given by

$$a = \frac{2\dot{\phi}}{\Lambda_3^9} \left(\frac{5G_{5X}}{X} + 2G_{5XX} \right) \quad (3.20)$$

$$b = -\frac{6\dot{\phi}^2}{\Lambda_3^6} \left(\frac{G_4}{X^2} - \frac{4G_{4X}}{X} - 4G_{4XX} \right) - 3M_{pl}^2 \quad (3.21)$$

$$c = -\frac{6\dot{\phi}}{\Lambda_3^3} G_{3X} \Lambda_2^4 X \quad (3.22)$$

$$d = V + \frac{\Lambda_2^4 X}{2} - G_2 \Lambda_2^4. \quad (3.23)$$

The general solution of the above cubic equation is

$$H = -\frac{b}{3a} - \frac{2^{3/2}(3ac - b^2)}{3a \left[9abc - 2b^3 - 27a^2d + (9abc - 2b^3 - 27a^2d)\sqrt{4(3ac - b^2)^3} \right]^{1/3}} + 2^{-3/2} \left[9abc - 2b^3 - 27a^2d + (9abc - 2b^3 - 27a^2d)\sqrt{4(3ac - b^2)^3} \right]^{1/3}. \quad (3.24)$$

According to the discussion of this subsection, the general bounce requirements are $H = 0$ and $\dot{H} > 0$ at the bounce point. Hence, using (3.24), the first requirement, namely $H = 0$, gives us the conditions

$$b^2 = 3ac; d = 0 \quad (3.25)$$

or

$$b^2 = 3ac; d = \frac{b^3}{18a^2}, \quad (3.26)$$

which must hold at the bounce moment. On the other hand, the second requirement, namely $\dot{H} > 0$, using (2.10) leads to the condition

$$\frac{\left(\Lambda_2^4 X + 2\Lambda_2^4 X G_{2X} + 2M_{pl} \ddot{\phi} X G_{3X} \right)}{(4X G_{4X} - 2G_4 - 1)} > 0, \quad (3.27)$$

around the bouncing point.

Let us make some comments on the conditions (3.25) or (3.26) and (3.27). As we observe, these conditions depend mainly on the functions G_I 's, however they depend on $V(\phi)$ too, since coefficient d in (3.25) or (3.26) includes $V(\phi)$, while $\ddot{\phi}$ that appears in (3.27) depends on $V(\phi)$ through the Klein-Gordon equation (2.13). Nevertheless, in the case where all G_I 's are zero there is no potential that can realize both conditions, which is expected since in the case of minimally-coupled general relativity the null energy condition cannot be violated and thus a bounce cannot be realized. On the other hand, in the case where the potential is zero, there are suitable G_I 's that can satisfy (3.25) or (3.26) and

(3.27) and induce a bounce. From these we deduce that the crucial ingredient of bounce realization is the Galileon functions G_I 's and not the potential $V(\phi)$. However, this does not forbid one to consider a non-zero potential in order to alleviate some tuning from the functions G_I 's, or in order to transfer all the required tuning from the G_I 's to the suitably reconstructed, complicated $V(\phi)$.

Let us consider the case where the potential is non-zero and free to be suitably reconstructed. Observing conditions (3.25) and (3.26), one can easily see that the simplest model of weakly broken galileon theories possible to generate a bounce must have the first three G_I functions non-zero, namely $G_2 \neq 0, G_3 \neq 0, G_4 \neq 0$ and $G_5 = 0$, since if $G_5 = 0$ then the condition $b^2 = 3ac$ cannot be satisfied if $G_4 = 0$. In this simplest model, at the bounce point we have $a = b = 0$ and thus (3.25), (3.26) imply that at the bounce point:

$$\dot{\phi}^2|_b = \frac{M_{pl}^2 \Lambda_3^6}{2 \left(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2} \right)} \quad (3.28)$$

$$V(\phi)|_b = G_2 \Lambda_2^4 - \frac{\Lambda_2^4 X}{2}. \quad (3.29)$$

Additionally, using the solution (3.24), we deduce that before the bouncing point ($H < 0$) we must have $b > 0$, while after the bouncing point ($H > 0$) we must have $b < 0$, or equivalently

$$\dot{\phi}^2 < \frac{M_{pl}^2 \Lambda_3^6}{2 \left(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2} \right)} \quad \text{for expansion} \quad (3.30)$$

$$\dot{\phi}^2 > \frac{M_{pl}^2 \Lambda_3^6}{2 \left(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2} \right)} \quad \text{for contraction.} \quad (3.31)$$

Let us apply these in the model (3.4), which indeed belongs to the subclass of simplest models considered here. In this case (3.28) becomes:

$$\dot{\phi}^2|_b = \frac{M_{pl}^2 \Lambda_3^6}{30}, \quad (3.32)$$

while (3.30), (3.31) become respectively

$$\dot{\phi}^2 < \frac{M_{pl}^2 \Lambda_3^6}{30} \quad \text{for expansion} \quad (3.33)$$

$$\dot{\phi}^2 > \frac{M_{pl}^2 \Lambda_3^6}{30} \quad \text{for contraction,} \quad (3.34)$$

and finally (3.27) reads as

$$\frac{\dot{\phi}^2 \Lambda_2^8 + 4\Lambda_2^4 \dot{\phi}^4 + 2M_{pl} \ddot{\phi} \dot{\phi}^2 \Lambda_2^4}{6\dot{\phi}^4 - \Lambda_2^8} > 0. \quad (3.35)$$

The most general form of $\dot{\phi}$ which satisfies (3.32), (3.34) and (3.35) is

$$\dot{\phi} = \alpha t^\gamma + \beta, \quad (3.36)$$

where $\gamma = 1, 3, 5, \dots$, $\beta = M_{pl}\Lambda_3^3/\sqrt{30}$ and α a negative constant. In order to give a simple example let us choose $\gamma = 1$. Integrating the above expression we obtain

$$\phi(t) = \frac{\alpha t^2}{2} + \beta t + \delta, \quad (3.37)$$

with δ an integration constant. Substituting (3.37) into the first Friedmann equation (3.5) we acquire

$$V(t) = 3H(t)^2 M_{pl}^2 - (t\alpha + \beta)^2 \left[\frac{1}{2} + \frac{3(t\alpha + \beta)^2}{\Lambda_2^4} + \frac{6H(t)(t\alpha + \beta)^2(15H(t) - \Lambda_3^3)}{\Lambda_3^6} \right], \quad (3.38)$$

Additionally, the second Friedmann equation (3.6) can provide the solution for $H(t)$. Hence, one can eliminate time, obtaining a general form of the potential $V(\phi)$ that generates a bouncing evolution.

Let us make a comment here on the role of the parameters Λ_2 and Λ_3 that characterize the weakly broken galileon invariance. As we mentioned above Λ_3 marks the scale suppressing the invariant galileon interactions, while the parameter Λ_2 , with $\Lambda_3 \ll \Lambda_2$, marks the scale suppressing the quantum-mechanically generated single-derivative operators [60], and thus in the limit where both Λ_2, Λ_3 go to M_{pl} weakly broken galileon invariance disappears, and the above theories become the usual covariant galileon ones. Hence, we can clearly see that the freedom to set the values of Λ_2, Λ_3 semi-independently makes the theories at hand different than usual covariant galileon ones, and the corresponding cosmology richer. In particular, one can see that the above bouncing requirements are much more difficult to be fulfilled in the case where Λ_2, Λ_3 are set to M_{pl} , and similarly the specific numerical examples of the previous paragraphs would be harder to be provided. We mention however that the comparison of the theories with weakly broken galileon invariance is made in relation to usual covariant galileon theories, and not with the general Horndeski theory, since as it was discussed in [60] the theories at hand fall within the class of general Horndeski construction.

We close this subsection by discussing on an issue that is present in principle in almost every bouncing scenario, namely the anisotropy issue. In particular, the bounce realization is in principle unstable against anisotropic stress, the so-called BKL instability [65], since the effective energy density in anisotropies ρ_{anis} evolves proportionally to $a(t)^{-6}$ and therefore in a contracting universe it increases faster than the matter and radiation energy densities, and thus the bounce should be realized in a non-isotropic and non-homogeneous spacetime.

However, although anisotropies grow faster than the background evolution, they will not dominate quickly, since this is related to the initial conditions of the anisotropy generation. Specifically, if the anisotropies arise from the backreaction of cosmological perturbations, which is of quantum origin, then the moment that the anisotropies will dominate over the background depends on the energy scale of the universe during matter contraction, which is typically at quite high energy scales. Hence, one can reliably consider an FRW background evolution up to the bounce phase.

Apart from this, there are many mechanisms that can be additionally introduced in order to ensure that even at high energy scales the anisotropies will not dominate and make

the universe depart from its FRW evolution. A well-studied case is to realize an ekpyrotic scenario, through the introduction of a negative exponential potential (see [63, 64, 66, 67] for the details of such a construction and [68, 69] for its observational confrontations), and thus it can be straightforwardly introduced in the present galileon scenario too (though one should be careful not to destroy the background bouncing solution, i.e. he should follow the procedure of subsection 3.1.2).

Having these in mind, in the following section, where we extract the observables of the bounce phase, we perform our analysis in the FRW geometry, without the need to examine a non-isotropic background evolution.

3.2 Reconstruction of cyclic evolution

Let us now extend the above analysis constructing a sequence of bounces and turnarounds, i.e. examining the realization of cyclic evolution. Without loss of generality we consider an oscillating scale factor of the form

$$a(t) = A \sin(wt) + a_c, \quad (3.39)$$

where $a_c - A > 0$ is the scale factor value at the bounce, with $A + a_c$ the scale factor value at the turnaround. In this case we apply the reconstruction procedure of the previous subsection, namely relations (3.5)-(3.10), in order to extract the solutions for $\phi(t)$ and $V(t)$, and thus obtain the re-constructed potential $V(\phi)$. Hence, this re-constructed potential will be the one that generates the cyclic scale factor (3.39). Note that the matter sector has to be considered in this case, hence we can assume it to be dust, namely with $p_m = 0$ and with $\rho_m = \rho_{mb}(a_c - A)^3/a^3$, with ρ_{mb} the value at the bouncing point.

In order to provide a concrete example we proceed to a numerical application of the above steps. In Figure 4 we present the potential $V(\phi)$ that is reconstructed from the given cyclic scale-factor form (3.39), according to the above procedure, in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$.

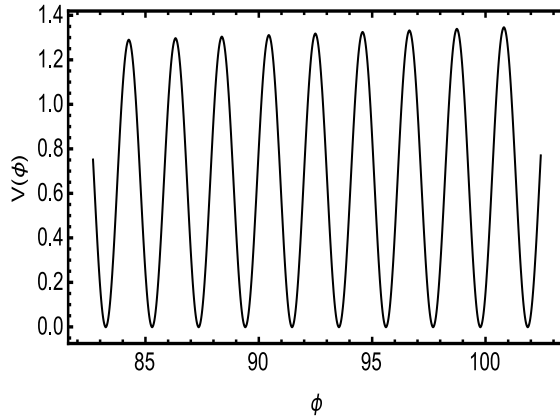


Figure 4. The reconstructed scalar potential $V(\phi)$ that generates the cyclic scale factor (3.39), in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. The model parameters have been chosen as $a_c = 0.01$, $A = 10^{-4}$, $w = 15$, $\rho_{mb} = 0.01$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

As we can see from Figure 4, in order to obtain a cyclic scale factor in the case where $G_2 = G_4 = X^2$, $G_3 = X$, we need a potential with an oscillatory form. Hence, we can now reverse the reconstruction procedure and consider a potential of the simple form

$$V(t) = V_1 \sin(w_V t) + V_2, \quad (3.40)$$

where V_1 , V_2 and w_V are parameters. As in the bounce reconstruction, inserting this form into Eqs. (3.5) and (3.8), we obtain a system of two ordinary differential equations for $a(t)$ and $\phi(t)$, that can be easily solved numerically. In Figure 5 we depict the scale factor $a(t)$ that results from the given potential (3.40). Thus, we indeed verify that the simple oscillatory potential (3.40) can generate a cyclic universe.

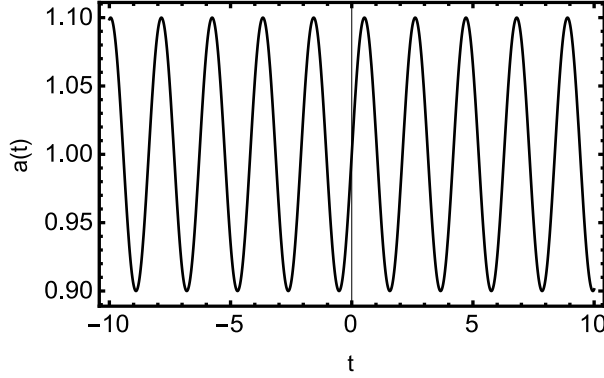


Figure 5. The evolution of the scale factor $a(t)$ that is generated by the simple oscillatory potential (3.40), in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. The potential parameters have been chosen as $V_1 = 1$, $V_2 = 0.1$, and $w_V = 3$, the matter energy density at the bounce as $\rho_{mb} = 0.01$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

Finally, we close this subsection by investigating some analytical cyclic solutions. A possible form of the scalar field ϕ which is able to satisfy the conditions at and around the bounce given by (3.32) and (3.34), and is also oscillatory in nature, reads as

$$\phi(t) = p \frac{\sin(wt)}{w} + \frac{st^2}{2} + tl + c_0, \quad (3.41)$$

where p , w , $s < 0$ and l are parameters and c_0 an integration constant. Substituting (3.41) either into (3.8) or into (3.5), we obtain

$$V(t) = 3H(t)^2 M_{pl}^2 - [1 + st + p \cos(wt)]^2 \left\{ \frac{6H(t)[1 + st + p \cos(wt)]^2 [15H(t) - \Lambda_3^3]}{\Lambda_3^6} + \frac{1}{2} + \frac{3[1 + st + p \cos(wt)]^2}{\Lambda_2^4} \right\}, \quad (3.42)$$

while (3.6) can give the solution for $H(t)$. Hence, one can eliminate time, obtaining the potential $V(\phi)$ that generates a cyclic evolution.

In this subsection we showed that at the background level the theories with weakly broken galileon symmetry can give rise to cyclic cosmology. However, we stress that such

a possibility has mainly a theoretical interest in order to reveal the capabilities of the scenario, since these cyclic scenarios will suffer from the problems of every cyclic evolution concerning perturbation-related observables, such as the spectral index. In particular, as it was shown in [70], in every cyclic cosmological model at each cycle fluctuations grow on super-Hubble scales during the contracting phase, and this induces a jump in the curvature power-spectrum spectral index n_s by $\delta n_s = -2$, and hence these models lose predictability. Thus, in order to consider these models as realistic, one should extend them and incorporate mechanisms that could alleviate this problem, for instance through a long dark-energy period before the turnaround of each cycle, as it was done in [71].

4 Cosmological Perturbations in the bounce phase

In subsection 3.1 we investigated the bounce realization in the framework of weakly broken galileon theories at the background level. In this section we proceed to the investigation of perturbations. Such a study is necessary in every bouncing scenario, since, similarly to inflationary cosmology, they will be related to observations.

The usual process for generating the primordial power spectrum in inflationary cosmology requires that cosmological fluctuations initially emerge inside the Hubble radius, then they exit it in the primordial epoch, and finally they re-enter at late times [72]. In bouncing cosmology however, the quantum fluctuations around the initial vacuum state are generated well in advance of the bouncing phase, and as contraction continues they exit the Hubble radius, since the wavelengths of the primordial fluctuations decrease slower than the Hubble radius. Definitely, when the universe passes through the bounce point the background evolution could affect the perturbations scale-dependence mainly in the UV, however the IR regime, which is responsible for the observable primordial perturbations related to the large-scale structure, will remain almost unaffected since at this regime the gravitational modification effects are very restricted [73–75]. Hence, one can study the primordial power-spectrum formation within standard cosmological perturbation theory.

Let us start by analyzing the perturbations in the framework of weakly broken galileon theories [60]. As usual, we consider that at linear order scalar and tensor perturbations decouple and evolve independently, and moreover note that for the present class of theories, which form a subclass of Horndeski theory, the equation of motion for the scalar field is still of second order. One novel feature of the present scenario is that apart from the usual symmetries present in FRW geometry, we additionally have the weakly broken galileon invariance. Hence, in the following we will see its effect on the perturbations.

We follow the usual Arnowitt-Deser-Misner (ADM) formalism, in which the metric is decomposed as

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j), \quad (4.1)$$

where $N = 1/\sqrt{-g^{00}}$ is the lapse and N^i the shift functions, while h_{ij} is the 3D metric on constant time hypersurfaces. In order to study the perturbations, we need to expand the action up to quadratic order in metric fluctuations. The intrinsic curvature of equal-time hypersurfaces, i.e. ${}^{(3)}R$, is at least linear in perturbations, while the extrinsic curvature of

equal-time hypersurfaces, defined as

$$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad (4.2)$$

where the covariant derivative ∇_i are taken with respect to h_{ij} , must be perturbed around the flat FRW background. Hence, we consider

$$\begin{aligned} N &= 1 + \delta N, \\ K_{ij} &= H h_{ij} + \delta K_{ij}. \end{aligned} \quad (4.3)$$

The perturbed action then reads [60, 76, 77]

$$\begin{aligned} S = \int d^4x \sqrt{\gamma} N \left\{ \frac{M_{pl}^2}{2} f(t) \left[{}^{(3)}R + K^{ij} K_{ij} - K^2 \right] - 2\dot{f}(t) \frac{K}{N} + \frac{c(t)}{N^2} - \Lambda(t) \right. \\ \left. + \frac{M^4(t)}{2} \delta N^2 - \hat{M}^3(t) \delta K \delta N - \frac{\bar{M}^2(t)}{2} (\delta K^2 - \delta K^{ij} \delta K_{ij}) + \frac{\tilde{m}^2(t)}{2} {}^{(3)}R \delta N \right. \\ \left. - \frac{\bar{M}'^2(t)}{2} (\delta K^2 + \delta K^{ij} \delta K_{ij}) + m_1(t) {}^{(3)}R \delta K + \dots \right\}. \end{aligned} \quad (4.4)$$

The terms in the first line corresponds to zeroth and first order perturbations, whereas the rest of the terms are second order in perturbations (we neglect terms giving rise to higher order perturbations). The time dependent coefficient $f(t)$ can be always removed through a conformal transformation and thus we set it to 1. The quantities $M^4(t)$, $\hat{M}^3(t)$, $\bar{M}^2(t)$, \dots , are the various effective field theory coefficients whose explicit forms will be fixed using the Horndeski Lagrangian [60]. As it was shown in [60], one finds that $\bar{M}^2 = \tilde{m}^2$, since only the combination $-\delta K^2 + \delta K^{ij} \delta K_{ij} + {}^{(3)}R \delta N$ appears in the action, which being a redundant operator can in turn be omitted by redefining the metric. Therefore, the only non-zero effective field theory coefficients are $M^4(t)$ and $\hat{M}^3(t)$.

In order to extract the equations for scalar and tensor perturbations, we work in the unitary gauge, which fixes the time and spatial reparametrization. In this gauge the metric and scalar field perturbations are given by [78]

$$\delta\phi = 0; \quad h_{ij} = a^2 e^{2\zeta} \delta_{ij}, \quad (4.5)$$

where ζ parametrizes the scalar fluctuations. In the following subsections we study scalar and tensor perturbations separately.

4.1 Scalar Perturbations

Working in the unitary gauge, setting all effective field theory coefficients (apart from $M^4(t)$ and $\hat{M}^3(t)$) to zero, and using the Hamiltonian and momentum constrain equations, one obtains the following quadratic action for the scalar perturbations ζ [60]

$$S_\zeta = \int d^4x \, a^3 A(t) M_{pl}^2 \left[\dot{\zeta}^2 - c_s^2 \frac{(\nabla\zeta)^2}{a^2} \right], \quad (4.6)$$

where

$$A = \frac{M_{pl}^2 \left(3\hat{M}^6 + 2M_{pl}^2 M^4 - 4M_{pl}^4 \dot{H} \right)}{\left(\hat{M}^3 - 2M_{pl}^2 H \right)^2}, \quad (4.7)$$

$$c_s^2 = \frac{\left(2M_{pl}^2 H \hat{M}^3 - \hat{M}^6 + 2M_{pl}^2 \partial_t \hat{M}^3 - 4M_{pl}^4 \dot{H} \right)}{\left(3\hat{M}^6 + 2M_{pl}^2 M^4 - 4M_{pl}^4 \dot{H} \right)}. \quad (4.8)$$

For the explicit expressions of the effective field theory coefficients in terms of $G'_I s$, X and ϕ in the general case, one may refer to [77]. For the purpose of this work it is adequate to use the approximate expressions of the two remaining non-zero effective field theory coefficients, namely M^4 and \hat{M}^3 , at cosmological backgrounds, which read as [60]

$$M^4 \sim \hat{M}^3 H \sim M_{pl}^2 H^2. \quad (4.9)$$

Following the analysis of the previous section, we again consider the ansatzes (3.4), namely $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. Nevertheless, even in this simple case, whether A and c_s^2 , which have a time-dependence, remain positive or not depends on the background solution, as can be clearly seen from (4.7) and (4.22).

Since c_s^2 in general depends on the background solution, one should explicitly check its positivity in any specific example. Concerning for instance the specific example of subsection 3.1.1, we insert the ansatz for the background bouncing scale factor into (4.7) and (4.22) and in Fig. 6 we depict the evolution of the sound speed square around the bounce phase. Indeed, for this specific example, c_s^2 (and A) is positive, and these features act in favor of stability.

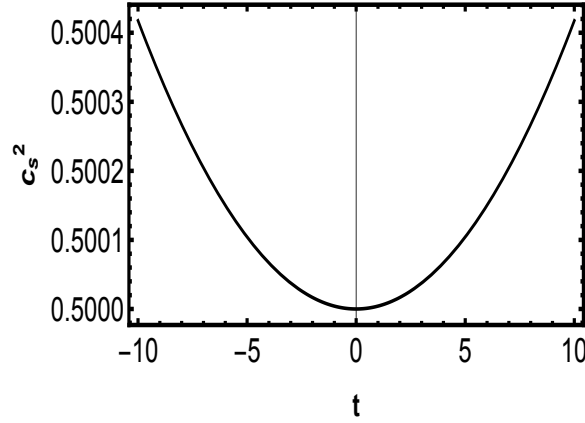


Figure 6. The evolution of the sound speed square, for the bouncing solution of subsection 3.1.1, i.e for the bouncing scale factor (3.1), in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$, and with $a_b = 0.2$, $B = 10^{-5}$, $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in M_{pl} units.

Proceeding forward, and in order to provide a well-defined perturbation quantization, we perform the usual Fourier transformation and introduce the canonical variable

$$\sigma_k = z\zeta_k; \quad z = a\sqrt{A}. \quad (4.10)$$

Thus, the equation of motion is given by

$$\sigma_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) \sigma_k = 0, \quad (4.11)$$

where primes represent derivatives with respect to conformal time $\eta = \int a^{-1}(t) dt$ [61]. Defining

$$M^2(\eta) = \frac{A'^2}{4A^2} - \frac{A''}{2A} - \frac{3HA'}{2A} - \frac{a''}{a}, \quad (4.12)$$

we can rewrite the above equation as

$$\sigma_k'' + [c_s(\eta)^2 k^2 + M^2(\eta)] \sigma_k = 0. \quad (4.13)$$

In summary, the above equation corresponds effectively to a massive scalar field, whose mass and sound speed square are time-dependent, and thus the solution will depend on the specific background evolution one imposes.

Let us now apply the obtained background bouncing solutions of the previous section in the above equation. As usual, we focus on the contracting phase far away from the bounce point, since this is the phase where the scale-invariant power spectra for curvature and tensor modes are obtained. In particular, for the contracting phase described by (3.1), and far from the bouncing point, where the scale factor evolves as

$$a(t) \approx t^{2/3} \approx \eta^2, \quad (4.14)$$

we obtain that

$$A \simeq M_{pl}^2, \quad (4.15)$$

$$c_s^2 \simeq 1. \quad (4.16)$$

Hence, equation (4.13) reduces to

$$\sigma_k'' + \left[k^2 - \frac{2}{\eta^2} \right] \sigma_k \simeq 0. \quad (4.17)$$

At early stages the k^2 -term dominates and hence the gravitational effects can be neglected. Therefore, since the scalar fluctuations effectively correspond to a free scalar propagating in a flat spacetime, we can consider that the initial condition acquires the form of the Bunch-Davies vacuum [79]:

$$\sigma_k \simeq \frac{e^{-ik\eta}}{\sqrt{2k}}.$$

Using these vacuum initial conditions we can solve the perturbation equation (4.17), acquiring

$$\sigma_k = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right). \quad (4.18)$$

Hence, we deduce that due to the gravitationally-induced term in (4.17), after exiting the Hubble radius the quantum fluctuations could become classical perturbations. Furthermore, the amplitude of the scalar perturbations will keep increasing until the moment t_{bp} in which the universe enters the bounce phase.

From the definition of the power spectrum we obtain that $\zeta \sim k^{3/2}|\sigma_k|$ is scale-invariant in the present scenario. Additionally, the explicit calculation leads to a primordial power spectrum of the form

$$P_\zeta \equiv \frac{k^3}{2\pi^2} \left| \frac{\sigma_k}{z} \right|^2 \approx \frac{H_{bp}^2}{48\pi^2 M_{pl}^2}, \quad (4.19)$$

where $H_{bp} = \sqrt{B/9}$ is the absolute value of the Hubble parameter at t_{bp} , i.e. when the bounce phase starts.

We close this subsection by mentioning that although we performed the perturbation analysis in a general way, in order to obtain the power spectrum we focused as usual in the contracting phase far away from the bounce, since this is where the scale-invariant power spectra for curvature and tensor modes are obtained. Definitely, in this regime the action becomes fully canonical, and that is why the analysis and the obtained power spectrum coincides with the standard results [35, 36]. Nevertheless, the general analysis is both necessary and interesting if one desires to address the evolution of perturbations through the bounce phase. For completeness, we accommodate this issue following [73–75].

If one desires to investigate the processing of perturbations through the bounce phase, instead of the variable (4.10) it proves more convenient to use the (gauge-invariant) co-moving curvature perturbation \mathcal{R} [73]. In this case, instead of (4.11) one can write the equation of motion for perturbations as

$$\mathcal{R}_k'' + 2\frac{y'}{y}\mathcal{R}_k' + c_s^2 k^2 \mathcal{R}_k = 0, \quad (4.20)$$

where

$$y^2 = a^2 \frac{M_{pl}^2 \left(3\hat{M}^6 + 2M_{pl}^2 M^4 - 4M_{pl}^4 \dot{H} \right)}{\left(\hat{M}^3 - 2M_{pl}^2 H \right)^2} \quad (4.21)$$

$$c_s^2 = \frac{2M_{pl}^2 H \hat{M}^3 - \hat{M}^6 + 2M_{pl}^2 \partial_t \hat{M}^3 - 4M_{pl}^4 \dot{H}}{3\hat{M}^6 + 2M_{pl}^2 M^4 - 4M_{pl}^4 \dot{H}}. \quad (4.22)$$

As it has been shown in [74], for the modes which are of interest today, i.e. in the regime $k \ll \mathcal{H}$, the relevant equation is

$$\frac{d\zeta'}{d\eta} + \frac{(y^2)'}{y^2} \zeta' = 0, \quad (4.23)$$

where ζ is the perturbation variable, equal to \mathcal{R} in the small- k limit, which is the limit we are interested in. One solution of (4.23) is

$$\zeta'(\eta) = \zeta'(\eta_i) \frac{y^2(\eta_i)}{y^2(\eta)}, \quad (4.24)$$

with η_i the initial time where the initial conditions are set. Therefore, we deduce that the evolution of perturbations depends on the evolution of y^2 , given by (4.21). Following [74], we will examine the behaviour of y^2 at three different regimes.

- Regime $2|H(t)| \gg |\hat{M}^3|$.

In the regime $2|H(t)| \gg |\hat{M}^3|$ relation (4.21) in M_{pl} units is simplified as

$$y^2 = a^2 \frac{(3\hat{M}^6 + 2M^4 - 4\dot{H})}{H^2}, \quad (4.25)$$

under the constraint $2M^4 + 3\hat{M}^6 \gg 4\dot{H}$. Inserting this expression into (4.24) we can obtain the evolution of perturbations through the bounce, as long as we insert the evolution for the model parameters given in [60], namely (4.15), for our particular background solution of subsections 3.1.1 and 3.1.2.

- Regime $2|H(t)| \ll |\hat{M}^3|$.

In the regime $2|H(t)| \ll |\hat{M}^3|$ relation (4.21) in M_{pl} units is simplified as

$$y^2 = a^2 \frac{(3\hat{M}^6 + 2M^4 - 4\dot{H})}{\hat{M}^6}, \quad (4.26)$$

(alternatively we could use the parametrization of [74] and rewrite the Hubble parameter as $H = \alpha \Delta t_B$, where Δt_B denotes the bounce duration). Similarly to the previous regime, we can obtain the evolution of perturbations through the bounce by inserting (4.15) for the particular background solution of subsections 3.1.1 and 3.1.2 into (4.21) and then into (4.24).

- Regime $2|H(t)| \approx |\hat{M}^3|$.

In this regime, in M_{pl} units, the denominator of (4.21) goes to zero leading to $y^2 \rightarrow \infty$. Thus, after a time interval t_f , the perturbations become constant on superhorizon scales. As it has been discussed in detail in [73, 74], the equation of motion seem to become singular in this regime, however this feature is an artifact of the Newtonian gauge and is removed applying the harmonic gauge. The bounce phase ends after the time t_f . Finally, the evolution of perturbations through the bounce is numerically obtained by inserting (4.15) for the particular background solution of subsections 3.1.1 and 3.1.2 into (4.21) and then into (4.24).

4.2 Tensor Perturbations

Let us now proceed to the investigation of tensor perturbations following [77]. As usual, we can neglect the scalar perturbations in (4.4). Working in unitary gauge the tensor perturbations read as

$$h_{ij} = a^2(t) e^{2\zeta} \hat{h}_{ij}, \quad \det \hat{h} = 1, \quad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2} \gamma_{ik} \gamma_{kj}, \quad (4.27)$$

where γ_{ij} , which parametrizes the tensor perturbation, is assumed to be traceless and divergence-free, namely $\gamma_{ii} = 0 = \partial_i \gamma_{ij}$.

Using the additional weakly broken galileon symmetry and setting all effective field theory coefficients, apart from $M^4(t), \hat{M}^3(t)$, to zero, we acquire the second order action for tensor perturbations as

$$S_\gamma^{(2)} = \int d^4x a^3 \frac{M_{pl}^2}{8} \left[\dot{\gamma}_{ij}^2 - \frac{1}{a^2} (\partial_k \gamma_{ij})^2 \right]. \quad (4.28)$$

Fourier transforming the above equation and working with the canonically normalized variable $v_k = M_{pl} \gamma_k / 2$, we obtain the equation of motion as

$$v_k'' + \left(k^2 - \frac{a''}{a} \right) v_k = 0. \quad (4.29)$$

Let us now apply the obtained background bouncing solutions of the previous section in the above equation. In particular, for the contracting phase described by (3.1), where the scale factor evolves as $a(t) \approx t^{2/3} \approx \eta^2$, equation (4.29) reduces to

$$v_k'' + \left(k^2 - \frac{2}{\eta^2} \right) v_k = 0, \quad (4.30)$$

whose exact solution is given by

$$v_k = \frac{e^{-ik\eta}}{\sqrt{2k\eta}} \left(1 - \frac{i}{k\eta} \right). \quad (4.31)$$

Hence, the primordial power spectrum of tensor fluctuations is also scale-invariant, however its magnitude is

$$P_T \equiv \frac{k^3}{2\pi^2} \left| \frac{\sigma_k}{z} \right|^2 \approx \frac{H_{bp}^2}{48\pi^2 M_{Pl}^2}, \quad (4.32)$$

which is of the same order of the scalar perturbation. Hence, we deduce that the bouncing scenario at hand suffers from the usual problem of all matter-like bounce models, namely that the tensor-to-scalar ratio $r \equiv P_T/P_\zeta$ remains of the order one (the scalar power spectrum is not additionally amplified as in inflationary realization). This high value is in significant disagreement with the observed behavior, which according to Planck probe [80] suggests that $r < 0.11$ (95% CL), while the combined analysis of the BICEP2 and Keck Array data with the Planck data requires $r < 0.07$ (95% CL) [81].

Note that the above disagreement with the data may be a consequence of a no-go theorem that shows that probably all matter-like bounce models would suffer from such difficulties in matching observations [74].

In order to accommodate with current observations, and as it is usual in bouncing scenarios, we must introduce a mechanism that can magnify the amplitude of scalar perturbations, and thus reduce the tensor-to-scalar ratio. For instance one can consider an additional light scalar field, as in the bounce curvaton-bounce [82], which can evade the aforementioned no-go theorem and enhance isocurvature fluctuations, and then give rise to a scale-invariant spectrum for the adiabatic fluctuations due to kinetic amplification. In particular, introducing a massless scalar χ and considering it to couple to the galileon field

ϕ as $g^2\phi^2\chi^2$, one can follow the procedure of [82] and deduce that the tensor-to-scalar ratio can be reduced to values $r \simeq 10^{-3}$.

Before closing this section, let us make a comment on the stability of the above bouncing solutions. In particular, there is a discussion in the literature whether there exists a no-go theorem that forbids stable non-singular cosmologies in Horndeski theory, as it was claimed in [83, 84], which could be evaded only extending to beyond-Horndeski constructions [85]. The proof of this theorem postulates that the involved galileon functions G_I 's are non-singular, and that a specific quantity related to the tensor perturbation remains finite at the bounce point (see Eq. (10) of [83]). Abandoning the first postulate allows for a stable non-singular bounce in the Horndeski class through galileon functions G_I 's that diverge at the bounce point, as it was shown in [86]. In our bouncing solutions obtained in the present work, one can show that the second postulate is bypassed, and thus Kobayashi's no-go theorem is evaded. Hence, the scenario at hand is free of ghost instabilities and therefore we obtain a well behaved model in terms of perturbations. Since this issue has a separate interest, that is related to the full Horndeski theory and not only to its specific subclass of theories with weakly broken galileon invariance, we are going to discuss it in detail in separate work [87].

5 Conclusions

We have investigated the bounce and cyclicity realization in the framework of weakly broken galileon theories. In this subclass of modified gravity one introduces the notion of weakly broken galileon invariance, which characterizes the unique class of gravitational couplings that maximally preserve the defining symmetry. Hence, the curved-space remnants of the quantum properties of the galileon allow one to construct quasi de Sitter backgrounds that remain to a large extent insensitive to loop corrections [60].

We studied bouncing and cyclic solutions at the background level, reconstructing the potential that can give rise to a given bouncing or cyclic scale factor. Then, reversing the procedure, we considered suitable potential forms that can generate a bounce or cyclic behavior. Additionally, for a zero or non-zero given potential, we reconstructed the forms of the galileon functions that give rise to a bouncing solution. Finally, we presented some analytical expressions for the requirements of bounce realization. As we showed, bounce and cyclicity can be easily realized in the framework of weakly broken galileon theories.

Having obtained the background bouncing solutions, we proceeded to a detailed investigation of the perturbations, which after crossing the bouncing point give rise to various observables, such as the scalar and tensor spectral indices and the tensor-to-scalar ratio. We calculated their values and we saw that the scenario at hand shares the disadvantage of all bouncing models, namely that it provides a large tensor-to-scalar ratio. Hence, we discussed about possible solutions, namely the possibility of introducing an additional light scalar which could significantly reduce the tensor-to-scalar ratio through the kinetic amplification of the isocurvature fluctuations. These features make the scenario at hand a good candidate for the description of the early universe.

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